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ABSTRACTS



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Cantor Connectedness of Uniform Spaces

Nikita Shekutkovski*, Zoran Misajleski†, Tatjana
Atanasova Pachemska‡

*SS Cyril and Methodius University, Faculty of Natural Sciences and Mathematics,
Institute of Mathematics, Skopje

†SS Cyril and Methodius University, Faculty of Civil Engineering, Department of
mathematics and Informatics, Skopje

‡Goce Delchev University, Faculty of Computer Sciences, Shtip

The first definition of connectedness was given by Cantor in the second half of 19th century.

We spread the Cantor definition to uniform spaces and show that it is equivalent to standard definition:

A uniformly connected space is a uniform space X such that every uniformly continuous function from X to a discrete uniform space is constant. A uniform space X is called uniformly disconnected if it is not uniformly connected.

By this, is justified to use the notion Cantor connectedness for uniform connectedness. We prove several properties of Cantor connectedness for uniform spaces, using the notion of chain.

Change of basepoint of the proximate fundamental group

Emin Durmishi*

*University of Tetovo, Faculty of Natural Science and Mathematics, Tetovo

For a covering \mathcal{V} of Y , the function $f: X \rightarrow Y$ between two topological spaces is said to be \mathcal{V} -continuous at $x \in X$ if there exists a neighborhood U_x of x and $V \in \mathcal{V}$ such that $f(U_x) \subseteq V$. The function is said to be \mathcal{V} -continuous if it is \mathcal{V} continuous at each point of the domain.

For a covering \mathcal{U} of the topological space X and $U \in \mathcal{U}$, the star of U is the set $\text{st}U = \cup\{x \in W \mid W \in \mathcal{U}, W \cap U \neq \emptyset\}$. By $\text{st} \mathcal{U}$ we denote the collection of all $\text{st}U$ sets, for each $U \in \mathcal{U}$.

The $\text{st} \mathcal{U}$ -continuous function $u: I \rightarrow X$ which is \mathcal{U} -continuous on ∂I is called a \mathcal{U} -path in X between $x = u(0)$ and $y = u(1)$. We prove that a topological space X is connected if and only if for any covering \mathcal{U} of X there exists a \mathcal{U} -path between any two points in X .

We apply this result on proving the change of basepoint theorem in the case of proximate fundamental group.

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The Hurwitz space of $PSL(5, q)$

Haval M. Mohammed Salih and Amin Mahamad
Zebari

Soran University, Faculty of Science, Erbil\Iraq

Let q be a prime power and $g \leq 2$. In this article, we assume that G is a finite group with $PSL(5, q) \leq G \leq Aut(PSL(5, q))$ and G acts on the projective points of 4-dimensional projective geometry $PG(4, q)$. We show that G possesses no genus g group if $q > 3$. Furthermore, the connectedness of the Hurwitz space $H_r^{in}(G)$ are studied for these groups that possess genus zero and one.

Topological direct sum of subspaces

Dragan S. Rakić* and Dragan S. Djordjević†

* University of Niš, Mechanical Engineering Faculty, Niš\Serbia

†University of Niš, Faculty of Sciences and Mathematics, Niš\Serbia

We define and investigate a topological direct sum of subspaces of a normed space X . Using a connection between this sum and the decomposition of the identity operator, we discuss the appropriate matrix form of a bounded linear operator on X . We emphasize the cases when X is a Banach or Hilbert space. The infinite topological direct sum of subspaces is considered as a particular case.

Proximate Approximative Retracts in Compact Metric Spaces

Abdulla Buklla

SS Cyril and Methodius University, Faculty of Natural Sciences and Mathematics,

Institute of Mathematics, Skopje

Retracts are fundamental concepts in topology that help in understanding the structure and properties of spaces. A retract of a topological space X is a subspace A with a continuous map (retraction) from X onto A that leaves A fixed. This notion is crucial as it provides insights into simplifying spaces while retaining essential topological features.

This paper introduces the concept of proximate approximative retracts (proxAR) in compact metric spaces. It discusses definitions and properties of various retraction types, including proximate retracts (PR), approximative proximate retracts (APR), weak retracts (WR), and approximative weak retracts (AWR) (for definitions, see [1]). The relationships between these retraction types and proxAR are explored.

References:

[1] Jacek Klisowski. A survey of various modifications of the notions of absolute retracts and absolute neighborhood retracts. *Colloquium Mathematicum* 46.1 (1982): 23-35.

Fuzzy SSPO-separation Axioms and a new form of fuzzy compactness

Shkumbin Makolli and Biljana Krsteska

University of Prishtina, Pristina.
St. Cyril and Methodius University, Skopje

In this paper, we introduce the concept of new separation axioms named fuzzy SSPO-separation axioms by using the fuzzy strongly semi pre-open sets and we also introduce and investigate properties of a new form of fuzzy compactness, α -SSPO compactness. We define and investigate the relation between fuzzy separation axioms, fuzzy pre-separation axioms, and different forms of fuzzy continuous mappings. We also investigate the existence of a countable base of fuzzy strongly semi pre-open sets, we define the concept of *SSPO* separability, the concept of α -*SSPO* Lindelof sets and examine their properties. With the concepts of fuzzy strong semi pre-continuity, *SSPO*-irresolute continuous mappings, and other forms of fuzzy continuity, we investigate the new concept of fuzzy compactness and its properties in regard to the mentioned mappings.

Directional Stockwell transform and its desingularization formula

Astrit Ferizi* , Katerina Hadzi-Velkova Saneva†

* University of Prishtina, Faculty of Mathematics and Natural Sciences, Prishtina

† Ss. Cyril and Methodius University in Skopje, Faculty of
Electrical Engineering and Information Technologies, Skopje

In this paper, we introduce and study the directional Stockwell transform (DST) as a hybrid of the directional short-time Fourier transform and the ridgelet transform. We prove an extended Parseval identity and a reconstruction formula for this transform, as well as results for the continuity of both the DST and its synthesis transform on the appropriate space of test functions. We also develop a distributional framework for the DST on the Lizorkin space of distributions. Additionally, a connection of the DST with the Stockwell transform and the Radon transform is established and a desingularization formula is provided.

The vector equilibrium problem for posets

Ljubiša D. R. Kočinac^{1,2}, and Ali Farajzadeh³*

¹State University of Novi Pazar

²Kyrgyz National University, Bishkek, Kyrgyzstan

³Razi University, Kermanshah, Iran

By using the notions of upper and lower order preserving set-valued mappings we establish some fixed point theorems in partially ordered sets equipped with the hull-kernel topology. Then we provide a solution of the vector equilibrium problem. The main results of this article can be viewed as the set-valued versions of some theorems of Tarski.

On sequence convergence in $(3,j)$ -metric spaces, $j \in \{1,2\}$

Dončo Dimovski¹, Pavel Dimovski² and Tomi
Dimovski³

¹Macedonian Academy of Sciences and Arts,

²Ss. Cyril and Methodius University, Faculty of Technology and Metallurgy, Skopje

³Ss. Cyril and Methodius University, Faculty of Mechanical Engineering, Skopje

In this article, we show that a convergent sequence in $(3,2)$ -metric spaces has a unique limit. We give several examples in $(3,1,\rho)$ -metric spaces and $(3,1)$ -metric spaces in which a convergent sequence has more than one limit. We obtain sufficient conditions for a sequence in $(3,1)$ -metric space to have a unique limit.

ICTA 2024

A Measure-Theoretic Perspective on Almost Periodicity

Daniel Velinov

Ss. Cyril and Methodius University in Skopje,
Faculty of Civil Engineering, Skopje

In this talk, we explore various classes of multi-dimensional almost periodic type functions and their generalizations in general measure, delving into their structural insights and applications. We introduce new classes of multi-dimensional (ρ, m') -almost periodic type functions, highlighting their structural properties and practical applications in abstract Volterra integro-differential inclusions within Banach spaces. Additionally, we discuss their applications in semilinear Volterra integro-differential inclusions, providing a comprehensive overview of their significance in mathematical analysis and beyond.

Chain connectedness in a family of sets

Zoran Misajleski[†], Emin Durmishi[‡]

[†]SS Cyril and Methodius University, Faculty of Civil Engineering, Department of
mathematics and Informatics, Skopje

[‡]University of Tetovo, Institute of Mathematics, Tetovo

In [1] the notion of a chain connected set in a topological space is given. In [2] the notion of a chain connected set in a family coverings space i.e. a space that consists of a set and a family of coverings of the set, is introduced.

In this paper we will define chain connectedness of a family of sets. If the union of the elements of the sets is considered as a set X , and family of the sets as a covering of the sets X , we obtain an one covering space. Two sets are chain connected in the family, if they contain two elements, respectively, which are chain-related in the one covering space.

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- [2] Z. Misajleski, E. Durmishi, A. Velkoska, Chain Connected Set in a Space, Proceedings of the Codema 2022 (2022), 35-42.

Higher order proximate groups

Aneta Velkoska

*Faculty of Communication Networks and Security, University of Information Science
and Technology St.Paul the Apostle - Ohrid

Using the intrinsic definition of shape based on proximate sequences for compact and all topological spaces based on proximate nets indexed by open coverings in the paper Shekutkovski et al. [1], we define proximate fundamental group. In this paper the proximate groups of higher order will be introduced.

References:

- [1] N. Shekutkovski, A. Velkoska, One Invariant of Intrinsic Shape, Filomat 29:10 (2015), 2185–21

Relations among G-metric spaces and $(3,j)$ -metric spaces and $[3,\Delta,j]$ -metric spaces, $j \in \{1,2\}$

Dončo Dimovski

Macedonian Academy of Sciences and Arts

We examine the relations among G-metric spaces and $(3,j)$ -metric spaces and $[3,\Delta,j]$ -metric spaces, for $j \in \{1,2\}$. We show that a G-metric is a 3-metric and is a $[3,\Delta,1]$ -metric, and that a G-symmetric is a $[3,\Delta,2]$ -metric. We give examples of: a G-metric that is not a $[3,\Delta,2]$ -metric; a $[3,\Delta,2]$ -metric that is not a G-metric; and a $[3,\Delta,1]$ -metric that is not a G-metric.

Optimized Graphene Production and Layer Determination Using Machine Learning-Enhanced Electrochemical Methods

Beti Andonovic¹, Viktor Andonovikj², Aleksandar T. Dimitrov¹

¹ Faculty of Technology and Metallurgy, Ss Cyril and Methodius University in Skopje

² Jožef Stefan Institute, Ljubljana

Graphene, a revolutionary material, has attracted significant interest for its amazing properties and potential applications. This research aims to design, develop, and optimize cost-effective, high-quality graphene production through electrochemical methods and determine its number of layers. Graphene samples were produced by electrolysis in molten salts using nonstationary current regimes.

The study focuses on optimizing graphene production by electrolysis in molten salts, utilizing constant and reversing cell voltage, as well as constant and reversing overpotential methods. Compared to other processes, this electrolysis method is simpler, more ecological, and economical. It offers control over parameters like electrolyte type, applied voltage, current density, temperature, and graphite type.

A tree-based Machine Learning (ML) model describes the relationship between these parameters and graphene quality. The model, trained with production process parameters as inputs and graphene quality as the target variable, uses expert-provided quality labels. The extracted rules from the ML model guide optimal graphene production, achieving high-yield, cost-effective results.

An equation-based model was also used to calculate theoretical 002 X-ray diffraction (XRD) peak intensities, enhanced by partitioning the 2θ interval for improved accuracy. The model curves fit well with the XRD intensities of the quality 3 graphene samples. The parameters used in the equation allowed for

ICTA 2024

determining the number of graphene layers. Results align with Raman spectra C-peak position values, indicating the graphene samples are few-layered.

This integrated approach of theoretical modeling, electrochemical methods, and machine learning enhances the precision of layer determination and improves the efficiency and cost-effectiveness of graphene production.

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Functions preserving Cantor connectedness

Nikita Shekutkovski

*SS Cyril and Methodius University, Faculty of Natural Sciences and Mathematics,
Institute of Mathematics, Skopje

In [2], several theorems and examples are stated, considering functions preserving connectedness and functions preserving path connectedness. In [1] is stated the following

Theorem : Suppose X is Hausdorff, locally path-connected and 1-countable, Y is Hausdorff, and $f: X \rightarrow Y$ a function. Then f is continuous iff f preserves compactness and f preserves path connectedness.

In this paper the notion of functions preserving Cantor connectedness is introduced, and discussed results similar to those in [1] and [2].

References:

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Topological Data Analysis and Decision Trees for Breast Cancer Detection using Thermal Images

Sijche Pechkova*, Lyudmyla Wenger[†]

*Faculty of Technology and Metallurgy ,

SS Cyril and Methodius University, , Skopje†

IPectus - <https://millawenger.wixsite.com/ipectus> Berlin

In our research, we propose an innovative method for analyzing thermal images for breast cancer detection using machine learning. By implementing features extracted from Topological Data Analysis (TDA), we aim to boost the diagnostic precision and consistency of thermal imaging. Our dataset consists of thermal images from breast cancer patients, from which we isolate essential topological characteristics that encapsulate the inherent geometric and structural details. Subsequently, these TDA features are incorporated into our decision tree model to attain a classification precision of 92%. This highlights the potential of merging TDA with machine learning algorithms in medical image analysis. Our results indicate that the addition of topological features can significantly enhance the capabilities of conventional imaging methods, establishing a robust foundation for early and precise detection of breast cancer.

Uniformly paracompactness of ordered uniform spaces

¹Bekbolot Kanetov, ²Dinara Kanetova and ³Makhabat Beknazarova

¹Kyrgyz National University named after J. Balasagyn,
bekbolot.kanetov.73@mail.ru

²Central Asian International Medical University, dinara_kg@mail.ru

³Jalal-Abad State University named after B. Osmonov,
maxabat.beknazarova@mail.ru

A triple is called an ordered uniform space if:

1. The uniformity generates on interval topology;
2. The uniformity has a base, each covering of which consists of intervals [1].

In this case the uniformity is called an ordered uniformity of the ordered space. Ordered topological space is understood as a linearly ordered set, endowed with the interval topology, and the interval is any convex subset which is open in the interval topology.

If the covering of an ordered space consists of intervals, then covering is called interval.

Let be an ordered uniform space.

An ordered uniform space is said to be (countable) uniformly paracompact if every of its (countable) open interval covering has uniformly locally finite open refinement. An ordered uniform space is said to be strongly uniformly paracompact if every of its open interval covering has uniformly star finite open refinement.

ICTA 2024

Theorem 1. For an ordered uniform space the following conditions are equivalent:

- 1) Ordered uniform space is strongly uniformly paracompact;
- 2) Ordered uniform space is uniformly paracompact.

Theorem 2. Any ordered uniform space is countable uniformly paracompact.

References

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